STAT 2593

Lecture 040 - Inference About the Slope Parameter

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Inference About the Slope Parameter

Learning Objectives

1. Understand the sampling distribution for the slope parameter estimate.
2. Use the sampling distribution to test hypotheses or form confidence intervals.

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  - The hypothesis of interest would be $H_0 : \beta_1 = 0$.
  - In general, we may wish to test $H_0 : \beta_1 = \beta$.
- As we have seen repeatedly, in order to test these hypotheses we need to know the sampling distribution.

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  - $\widehat{\beta}_1 \sim N(\beta_1, \sigma^2/S_{xx})$.
  - When σ^2 is not known, we can replace it with s^2.

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- From this sampling distribution, we can use the t distribution to derive hypothesis tests and confidence intervals.

Summary

- The sampling distribution for the slope parameter will, under normality assumptions, be a t distribution with $n-2$ degrees of freedom.
- This sampling distribution can be used to test hypotheses or form confidence intervals exactly in the same way that we did for the mean.